Efficient Control of Floods with Gated Reservoirs on Short Timescales

with reference to the Lee Dams and the flooding of Cork in November 2009

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This poster compares two procedures, 'lake' control' and 'e-target control', for the operation of gated reservoirs protecting a downstream city from flooding during a single isosceles flood event (peak P [m^3/s] , duration T [s], volume V=P*T/2 [m^3]). 'Lake' control is local in time, always releasing a fraction of the flood-pool contents per unit time - in the simplest case, a constant fraction, the so-called 'linear' reservoir. In marked contrast, 'e-target' control always releases an outflow as close to e [m^3/s] as possible. When the inflow is less than e, and the flood pool is empty, the release is the inflow itself, as if the reservoir did not exist. When the inflow rate exceeds e, the excess is stored in the flood-pool, accumulating to a maximum stored volume S [m^3]. The flood-pool is returned to empty at a release rate e. The technical efficiency of an operating procedure in attenuating a reference flood can be measured in two equivalent ways: (1) as the smallest upper bound on the flow through the city, e, that a flood-pool volume S provides, or (2) the minimum flood-pool volume, S, that defends the city against a maximum flood flow, e.

We prove the following efficiency relationships

'Lake' control:	S/V ≈ 1 – e/P,	e/P ≈ 1 – S/V	least efficient
'e-target' control:	S/V = $(1 – e/P)^2$,	e/P = 1 – √(S/V)	most efficient

'e-target' control is almost twice as efficient as 'lake' control. We extend these results to a cascade of two reservoirs in series, with synchronous lateral inflow above and below each dam in proportion to upstream catchment area. There is almost no loss of efficiency under 'e-target' conjunctive control. The City of Cork provides an illustration: there would have been no inundation of its Central Island in Nov 2009 had 'e-target' control been in operation at its two dams; and '14km x 1.4m defensive walls' are not required to defend it.

Reservoir operating procedures during a flood event

'Lake' control

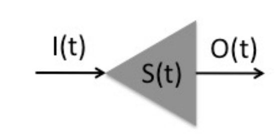
- Releases proportional to volume in storage
- "Don't worsen nature" *Nollumus mutari*

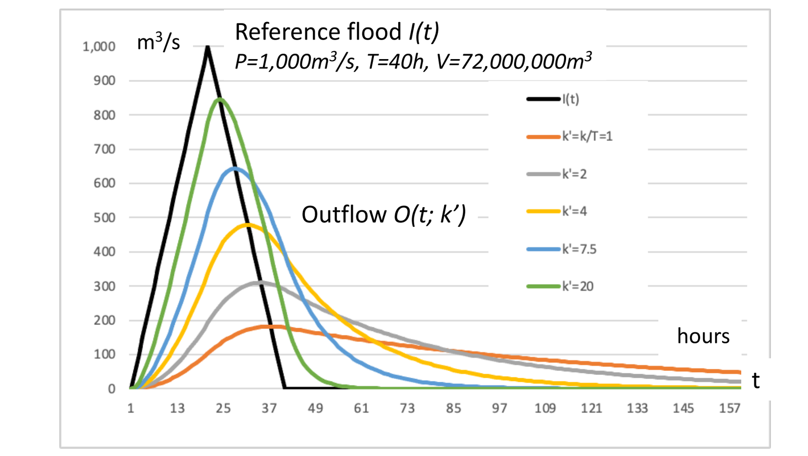
'e-target' control

- Release all inflows less than *e [m^3/s]*, as if the dams were not there
- Follow the Supreme Court ruling to protect the city "in certain circumstances": when the inflow exceeds e, store the e-excess in the flood-pool, throttle releases for tributaries below the dams, balance the dams with the 'empty-space rule', and empty the flood-pool at a rate *e*.

'Lake' control: outflow and storage

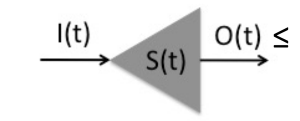
Release *O(t) = k*S(t)*
Outflow and Storage reach maxima, when *I(t") = O(t")* at time *t"(k')*.

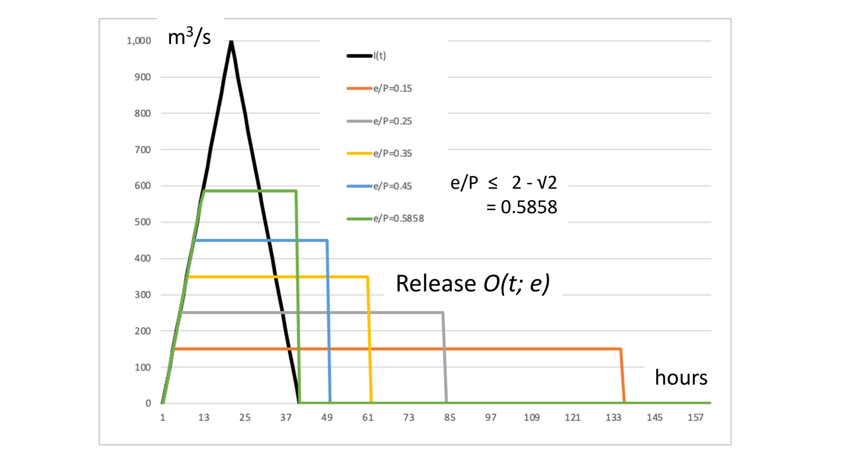




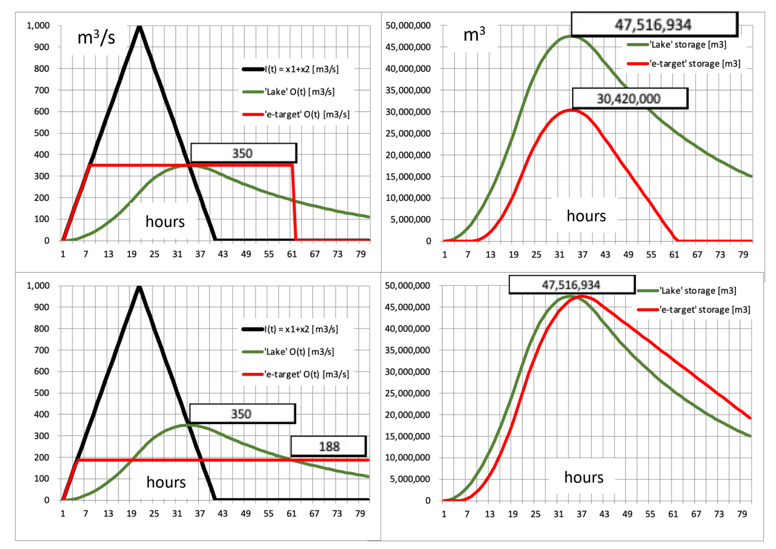
'e-target' control

Pass the inflow I(t) < e,
store the e-excess I(t) – e > 0
empty the flood-pool at a rate e





'e-target' control of the reference flood is almost twice as efficient as 'lake' control



Efficiency of 'e-target' and 'lake' control

of an isosceles flood: peak P, duration T, volume V=P*T/2

'e-target' control

S"/V = $(1 – e/P)^2$
e/P = 1 – √(S"/V)

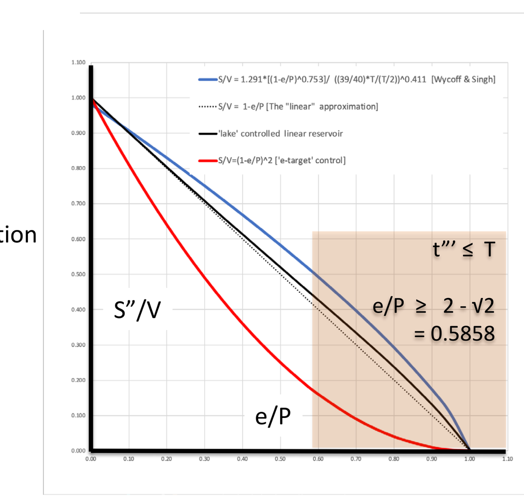
'lake' control

1.Transcendental parametric solution [Dooge, 1956]

2. Approximate solution : linearly increasing release rate [Wycoff & Singh, 1976] :

S"/V ≈ 1 – e/P
e/P ≈ 1 – S"/V

3. Wycoff & Singh regression study.



Cork conclusions 1-3: changing to 'e-target' control => no need for 14km x 1.4m walls in Cork

	1	2	3
Reference flood - 1/180 years P=1,000m^3/s. V=72,000,000m^3 at the Waterworks Weir	'lake' control no defenses pre-1990	'lake' control no defenses post-1990	'e-target' control no defenses post-1990
'No-flood-in-the-city' - m3/s	350	350	350
Storage volume required - m3	51,071,460	51,071,460	30,420,000
'No-flood-in-City'?	No-flood-in-City	City fooded	No-flood-in-City
flood-pool volume	24,604,327	17,109,506	17,109,506
hydro-pool volume	19,552,522	19,552,522	19,552,522
water-supply pool	7,865,946	7,865,946	7,865,946
total	52,022,795	44,527,974	44,527,974
Take from hydro-power-pool	19,552,522	19,552,522	13,310,494
Take from water-supply-pool	6,914,611	7,865,946	0
% take from hydro-power pool	100%	100%	68%
% take from water-supply pool	88%	100%	0%

Mathematical appendix

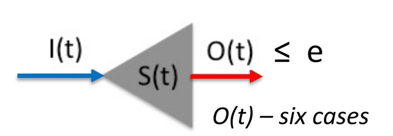
The following theorems confirm and clarify the results obtained from CC-SS-Lee (Conjunctive Control and Simulation System for the Lee), reported in the paper "Protecting the City of Cork from Flooding" presented via Zoom at the IHP/ICID National Hydrology Conference in November 2020: https://hydrologyireland.ie/wp-content/uploads/2020/12/08-OKane-Protecting-the-City-of-Cork-from-Flooding_merged_final.pdf, and also on the website www.savecorkcity.org

References
Dooge, James C.I. (1956) "Synthetic Unit Hydrographs based on triangular inflow. Appendix C Routing a triangular inflow." MS thesis, State University of Iowa, USA. June. 103 pages. Pages 92-94.

Wycoff, Ronald, L. and Singh, Udai P. (1976) "Preliminary hydrologic design of small flood detention reservoirs" Water Resources Bulletin, American Water Resources Association. Vol. 12, No.2. April. Pages 337 – 349.

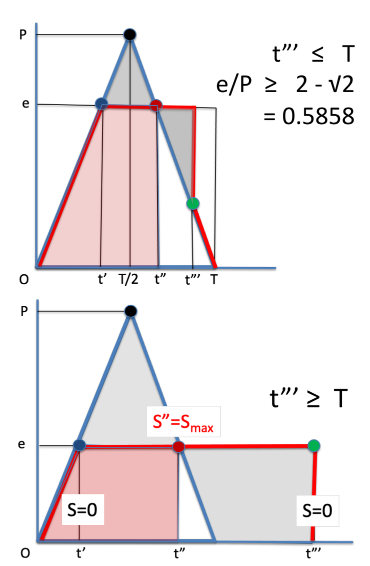
'e-target' control of isosceles flood

Pass the inflow I(t) < e,
store the e-excess I(t) – e > 0
empty the flood-pool at a rate e



	I(t)=0	0<I(t)<e	I(t)>e
S(t)=0	O(t)=0	O(t)=I(t)	O(t)=e
S(t)>0	O(t)=e	O(t)=e	O(t)=e

dS/dt = I - O	I(t)=0	0<I(t)<e	I(t)>e
S(t)=0	0	I(t) - I(t)=0	I(t) - e
S(t)>0	0 - e	I(t) - e	I(t) - e



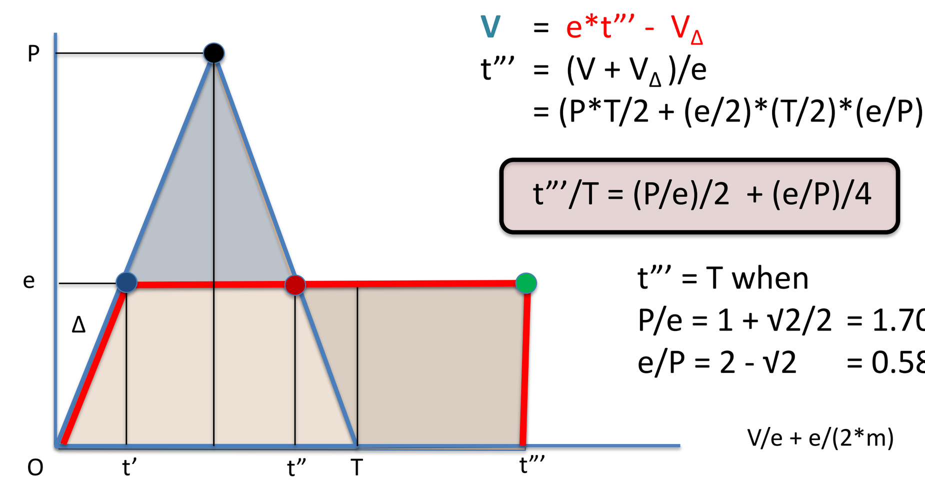
'e-target' control of isosceles flood

Case: duration t"'≥ T, peak P, base T, volume V=P*T/2

Inflow = outflow
V = e*t"' - $V_Δ$
t"' = (V + $V_Δ$)/e
= (P*T/2 + (e/2)*(T/2)*(e/P))/e

t"'/T = (P/e)/2 + (e/P)/4

t"' = T when
P/e = 1 + √2/2 = 1.7071,
e/P = 2 - √2 = 0.5858



Five-part quadrature of an isosceles flood I(t) in a reservoir under 'e-target' control, t"'≥ T

peak P, duration T, volume V=P*T/2, slope m=2*P/T, target e

S(t) = [S0(t), S1(t), S2(t), S3(t), S4(t)], S0(0) = 0,

S0(t) =	0	0 ≤ t ≤ t'
S1(t) =	$m*t^2/2 - e*t + e^2/(2*m)$	t'≤ t ≤ T/2
S2(t) =	$- m*t^2/2 + (m*T- e)*t - V + e^2 /(2*m)$	T/2 ≤ t ≤ T
S3(t) =	$- e*t + V + e^2 /(2*m)$	T ≤ t ≤ t"'
S4(t) =	0,	t"'< t

where t'= e/m=(T/2)*(e/P)), t"' = V/e + e/(2*m) = (T/2)*[(P/e) + (e/P)/2]

The time-to-peak storage S"= S_{max}= max S2(t) occurs when m*(T-t") = e :
t"/T = 1 - (e/P)/2

The ratio of flood storage and volume S"/V, and the attenuation ratio e/P, satisfy
S"/V = $(1 – e/P)^2$, e/P = 1 - √(S"/V)

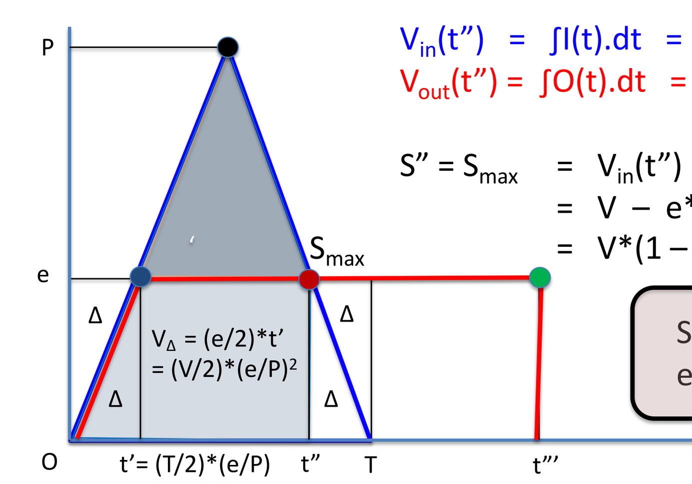
'e-target' efficiency theorem

relationship between attenuation (e/P) and storage (S"/V)
reference flood: peak P, base T, volume V=P*T/2

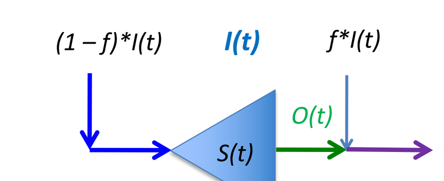
V_{in}(t") = ∫I(t).dt = V – $V_Δ$
V_{out}(t") = ∫O(t).dt = e*T – 3* $V_Δ$

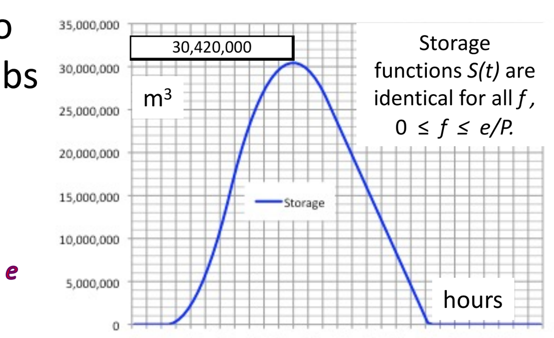
S" = S_{max} = V_{in}(t") – V_{out}(t")
= V – e*T + 2* $V_Δ$
= V*(1 – 2*(e/P) + $(e/P)^2$)

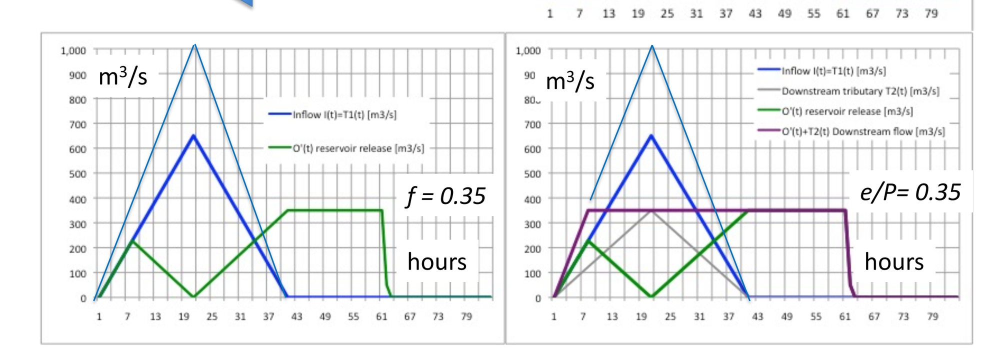
S"/V = $(1 – e/P)^2$
e/P = 1 – √(S"/V)



Throttle the release *O(t)* to 'store' the downstream tribs

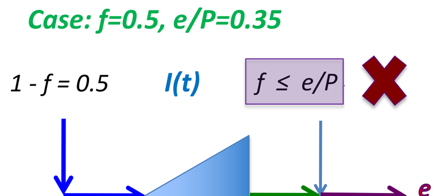


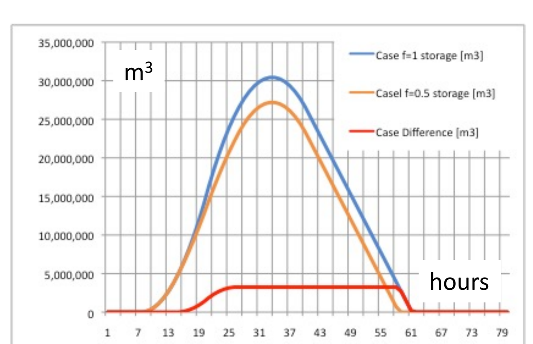


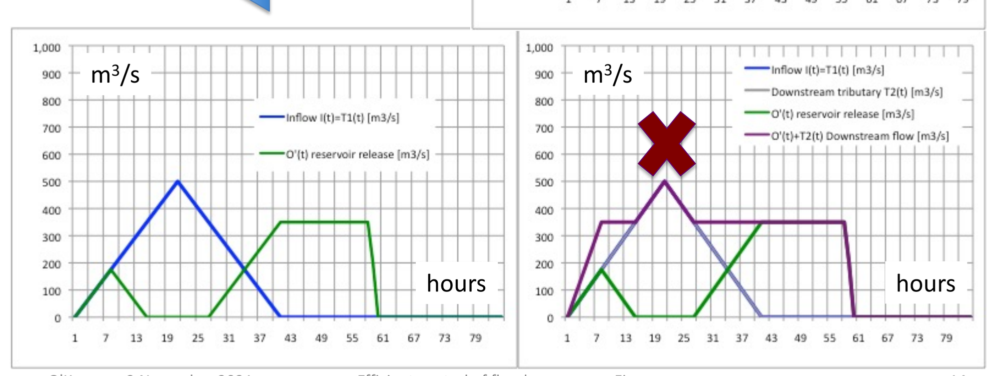


Throttle the release

Case: f=0.5, e/P=0.35

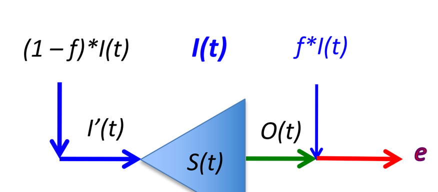






'e-target' throttle theorem

The storage function *S(t)* in the ODE *dS/dt = I'(t) – O(t)* is independent of *f*, the fraction of the flood below the dam, when *O(t)≥0 => f ≤ e/P.*

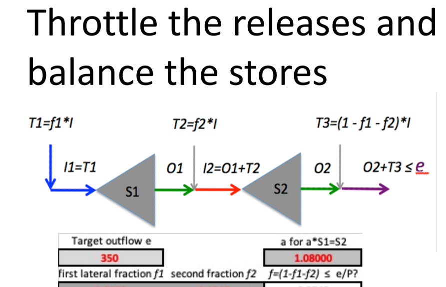


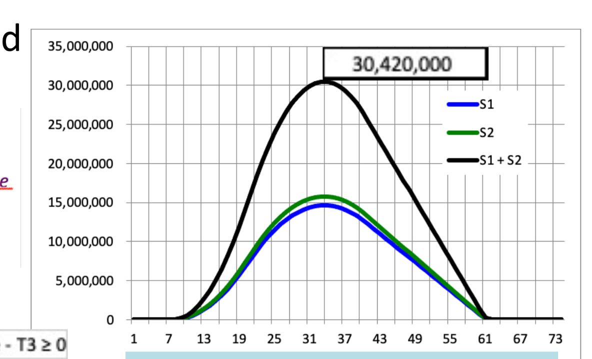
*I'(t) = (1 – f)*I(t)*
I'(t) - O(t) : six cases
f cancels !

	I'(t)=(1-f)*I(t)=0	I'(t)=(1-f)*I(t)<e	I'(t)=(1-f)*I(t)>e
S(t)=0	O(t) = 0	O(t) = (1-f)*I(t)	O(t) = e - f*I(t)
S(t)>0	O(t) = e	O(t) = e - f*I(t)	O(t) = e - f*I(t)

dS/dt = I' - O	I'(t)=0	I'(t)<e	I'(t)>e
S(t)=0	0	(1-f)*(I(t) - I(t))=0	(1-f)*I(t)-(e-f*I(t)) =
S(t)>0	0 - e	I(t) - e	I(t) - e

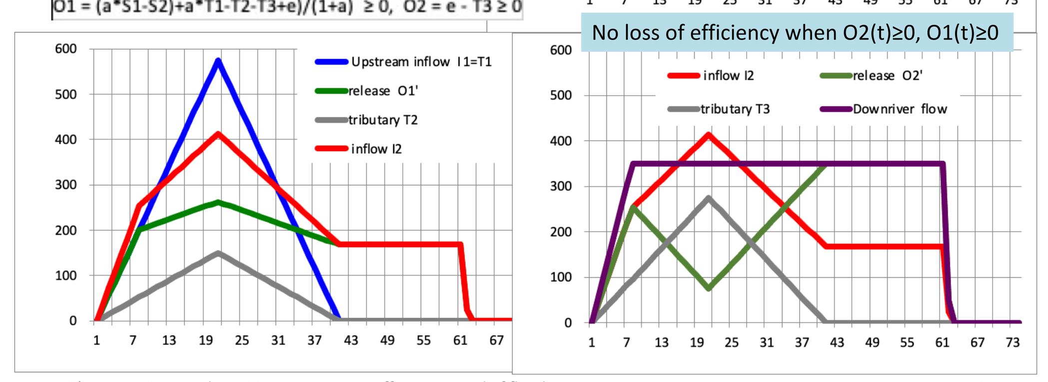
Throttle the releases and balance the stores





O1 = (a*S1-S2)+a*T1-T2-T3+e)/(1+a) ≥ 0, O2 = e - T3 ≥ 0

No loss of efficiency when O2(t)≥0, O1(t)≥0



Isosceles flood I(t) routed through a linear reservoir

peak P, duration T, volume V=P*T/2, slope m=2*P/T, parameter k

The solution S(t) of the 'lake' linear differential equation

dS(t)/dt = I(t) – O(t), O(t)=k*S(t), S(0)=S_0=0

for a continuous isosceles inflow function I(t) = [m*t, m*(T-t), 0] is S(t) =[S1(t), S2(t), S3(t)] = [O1(t), O2(t), O3(t)]/k where the disjoint outflow functions [O1(t),O2(t),O3(t)] are

O1(t) =	m*t	+ (m/k)*[exp(-k*t) - 1],	0 ≤ t ≤ T/2
O2(t) =	m*(T- t)	+ (m/k)*[1 + (1-2*exp(k*T/2))*exp(-k*t)],	T/2 ≤ t ≤ T
O3(t) =	0	+ (m/k)*[1 + exp(k*T) - 2*exp(k*T/2)]*[exp(-k*t)],	T ≤ t

Max O(t)/k = Max S(t) = S(t") = S" occurs at time T/2 ≤ t"≤ T when I(t")=O(t"), dS(t")/dt=0 :

t"/T = (1/(k*T))*Ln[(2*exp(k*T/2) - 1)]

The parametric relationship between flood attenuation e/P and storage S"/V is approximately linear: S"/V ≈ 1 – e/P

e/P = 2*(1 - t"/T) , S"/V = [4/(k*T)]*(1 - t"/T)

'Lake' control approximation theorem

linear release rate O(t) from 0 to e at t=t"=T-t'

Triangle Tr on base T, height e:
V_{Tr} = e*T/2 = V*(e/P)

V_{in}(t") = ∫I(t).dt = V – $V_Δ$
V_{out}(t") = ∫O(t).dt = V_{Tr} – $V_Δ$

S" = S_{max} = V_{in}(t") – V_{out}(t")
= V – V_{Tr}
= V*(1 – e/P)

S"/V = (1 – e/P)